Fluid Mechanics
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Lecture - 04
Concepts of Hydrostatic

Welcome all of you for this lecture on fluid mechanics. Today we will discuss about fluid statics that means fluid at rest.

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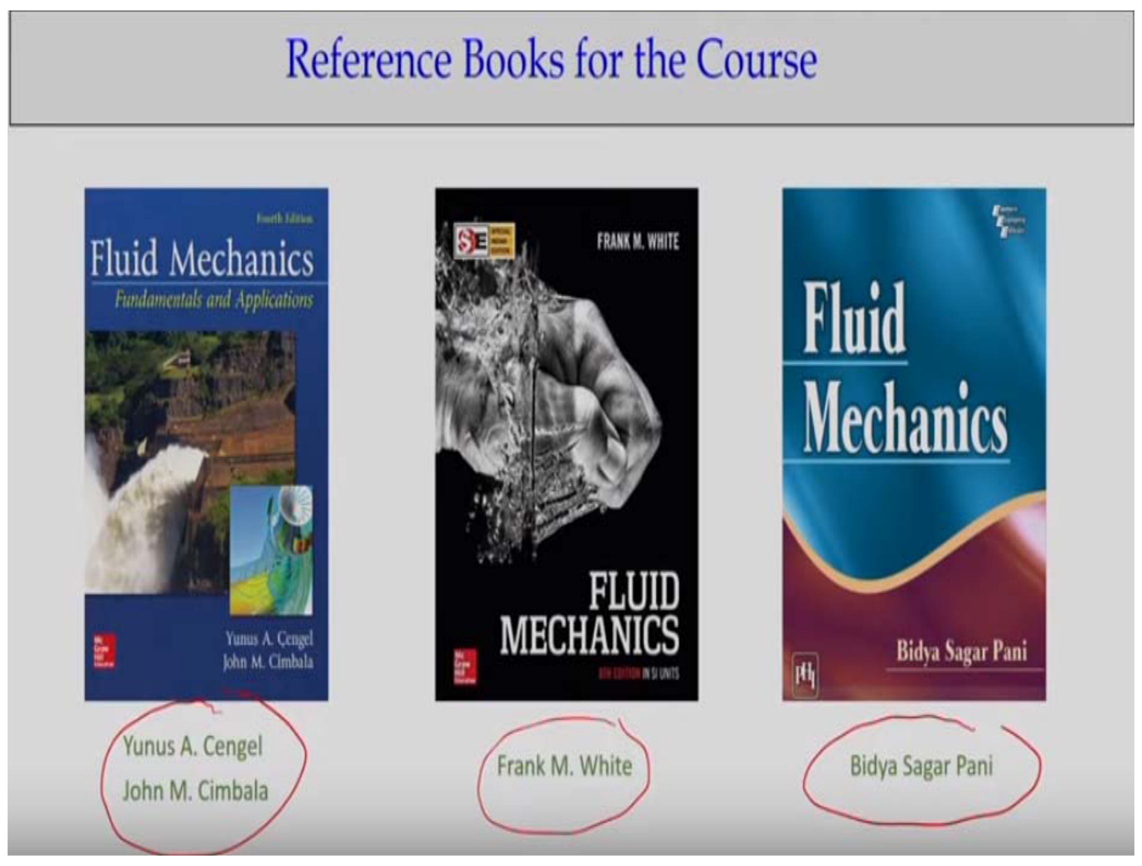

Before starting this lecture, again I want to repeat the reference books, like the most illustrations books like the Cengel Cimbala book which is very good book in terms of lot of illustrations, the examples, and the exercise what is given in Cengel Cimbala book. And who needs to very concise they can read it F.M. White book which gives mathematical forms with very classical problem solvings at exercise levels also the in examples problems.

And another books by Bidya Sagar Pani on fluid mechanics is concise which in Indian context book is very concise to read it. So please refer this reference books, the Fluid Mechanics the Fundamental and Applications by Cengel Cimbala or Fluid Mechanics F.M. White, or Fluid Mechanics by the Bidya Sagar Pani. So these are what the reference books I have been following it. So please follow these books for additional knowledge on this fluid mechanics course.

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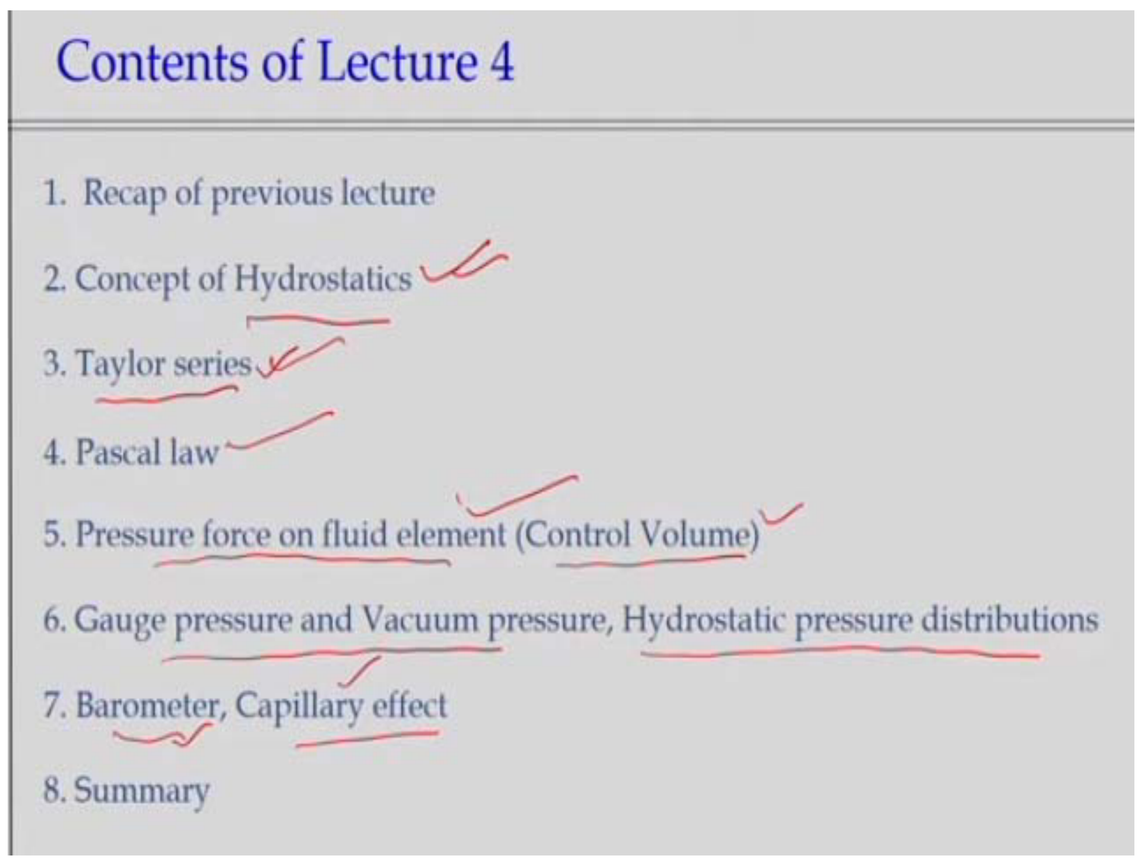

Now let us come to the contents of the today lectures. The first I will discuss what we so far in the last three lectures we discussed it. Then we will talk about concept of hydrostatics okay that means the fluid at the rest and most of the as you know it that when you consider a function okay you can approximate the function using a Taylor series, that the concept we will talk about. The Pascal law we will talk about.

Then the major components like the pressure force on fluid element, that is what is the control volumes. That what we will also discuss it. Then we will discuss about what is a gauge pressure, what is a vapor pressure, and hydrostatic pressure distributions. And there are two applications of this hydrostatic pressure distributions. One is barometer another is capillary effect. Then I will conclude today lectures by summarizing the lecture content.

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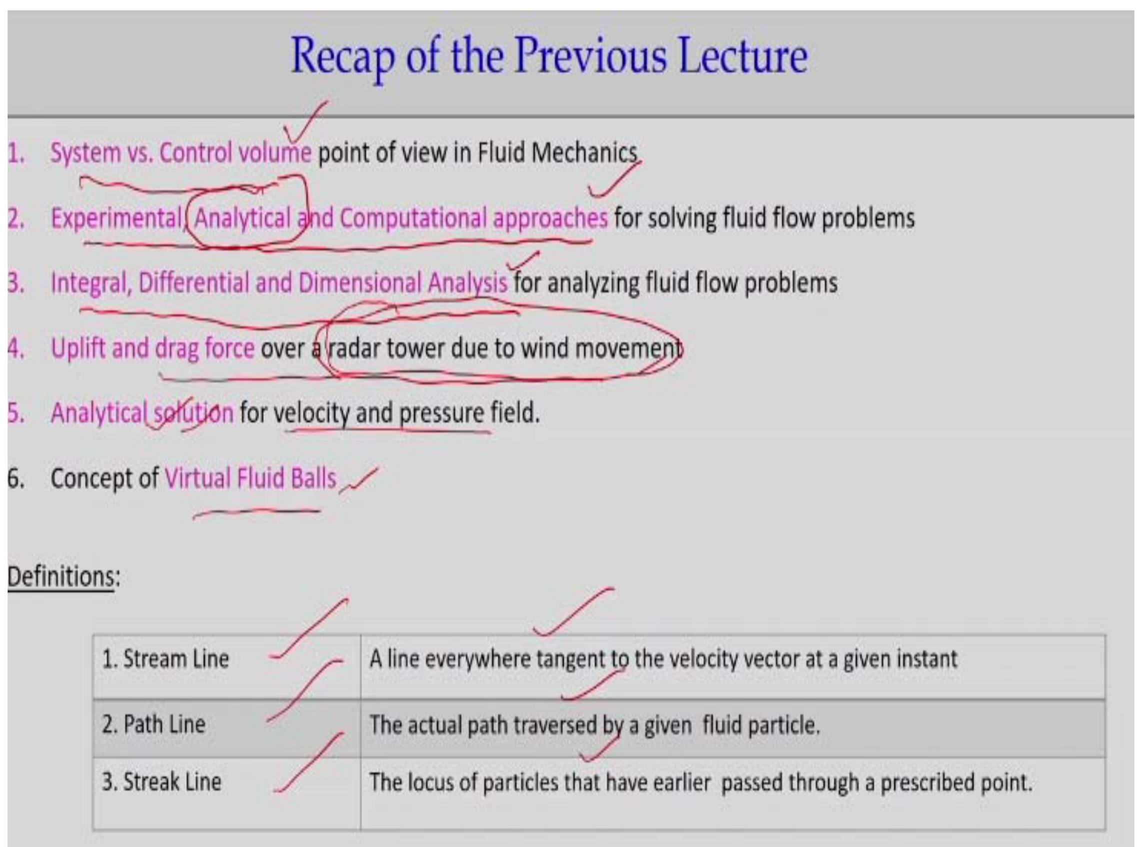

Now let us recap it, as of now what we learnt it. We already know it we talk about a either a system approach or the control volume approach to solve the fluid mechanics problems. And whenever you solve the fluid mechanics problems as I said it in the last class, we generally look for three velocities three fields, velocity field, pressure field and the density field.

But when fluid is incompressible, then we just look for the pressure field and the velocity field. So these two fields we can get it using this three different approaches as I discussed earlier. One is experimental method, conducting the experiment in a wind tunnel, computational approach, which is a computational fluid dynamic now extensively used for a very complex fluid flow problems or this analytical approach with a very simplified problem we can solve analytically to get the gross approximations of this pressure field the and the velocity field.

So that is what we look it for the flow which is considered incompressible flow. And then we already discussed about that to adopt these three different approaches we have to follow integral approach, differential approach and the dimensional analysis. And very interesting examples what I have given in the last class is that the how the force, very complex flow fields we get it in a radar tower due to this wind movement.

That is what we discuss it that and there is a option is there, we can do analytical solutions for that to get a velocity and pressure fields for a simple problem which can you solve it. And as again I can repeat it that the concept of virtual fluid balls we discussed lot and that what we will apply when you go for fluid kinematics and the fluid dynamics part which is later on I will discuss it that.

And as a definition already we said that what is the streamline, the pathline and the streaklines which is considered to define what type of flow is happening it and understanding or visualizing the fluid flow problems because that what using the streamlines, pathline and the streaklines.

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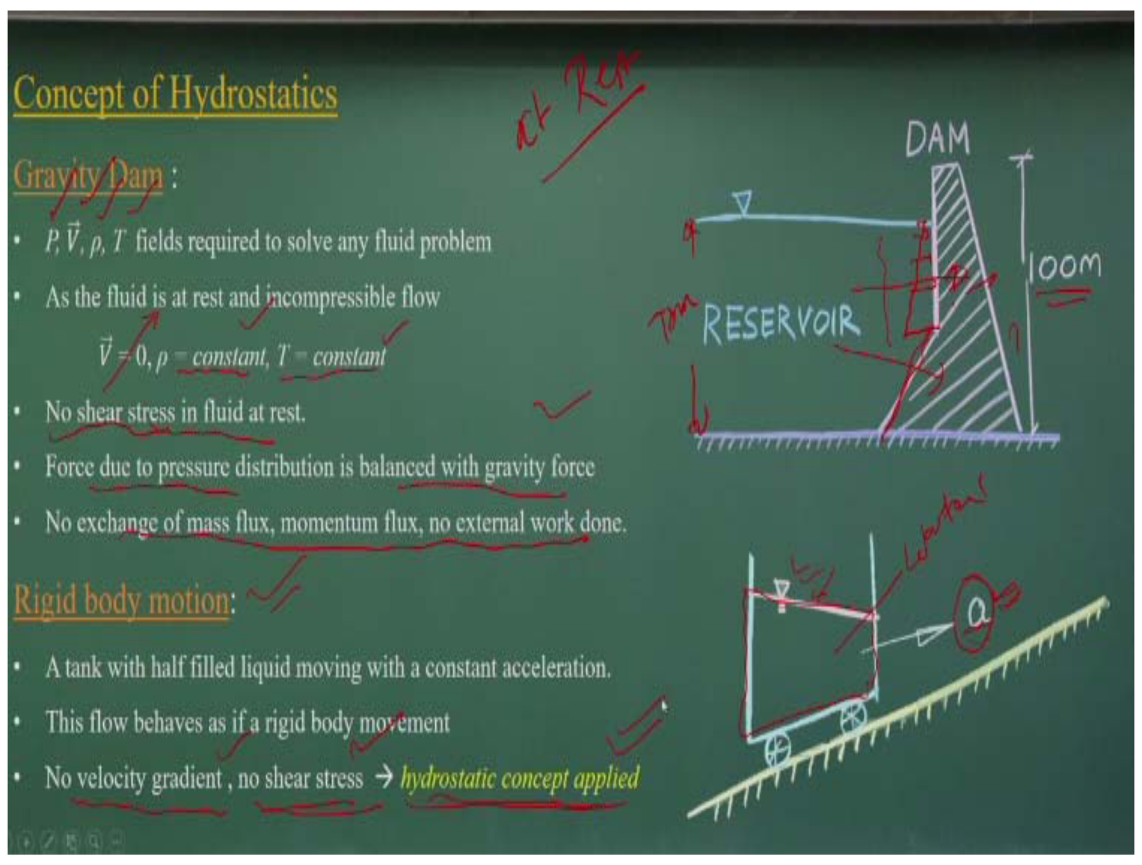

Now let us come to the very basic concept what we are talking about the fluid at rest. So the basically we are talking about now, the fluid at rest, okay? If it is a fluid is at rest, it is a very simplified problem now. Like as I said it any fluid flow problems we look at either the pressure field, the velocity field, or the density and temperature field.

When the fluid is rest now, very simply way the velocity vectors becomes zero and if I consider incomprehensible the density is a constant and if the temperature is not very much I need not need a thermodynamics first laws to define the problems. Then only the left is that the pressure field. That means when fluid is at rest conditions only we need to know it how the pressure variations is going over these fluid domains. That is what the simplified problems.

Since there is no velocity, there is no velocity gradient, definitely as Newton's second law says that there is no velocity gradients that means no shear stress. So when a fluid is at rest, there is no shear stress acting on that. So you can take a surface or take a control volume. Over that control surface you can define it the shear stress components become zero.

That is very simplified now that any control volume you consider it over that control volume surface as the fluid is at rest conditions, there is no shear stress acting on the control surface. So that is the very easy problems what we have now. And what the two forces we have? The gravity force and the force due to the pressure distribution. So the whatever the pressure distribution forces that what is equate with the gravity force.

Very simple things now, and since is a fluid is at the rest, so you can say that there is no mass flux is coming into the any control volumes or the momentum flux or no external work done it. So this is the what the simplified case. Like for example, if you take this dam, which is 100-meter-high and we have a reservoir, let you consider this is what 90 meter height from the bottom.

That is what the water levels is 90 meter from the bottom. And you can understand it because of these in the reservoirs the fluid is at the rest conditions and that rest condition exerting the pressures on these surface. So there is a vertical surface, there is a inclined surface. So we need to know it what is the fluid pressure is going to act on this dam, on these vertical surface and also the inclined surface.

Based on these the pressure load we can design this dam structure. That is what the examples for fluid at the rest. Similar way if you have an oil tanker, so we need to know it what is the pressure, the fluid oils with are in a rest how much of pressure is exerting on the wall of the tank.

So it is the problems what we use to design a tank, big reservoir tank or the natural reservoirs like a dam, we try to know it how much of pressure is going to act because of this fluid rest, fluid is at rest condition and how much of pressure is required. Because of this pressure and what is the pressure force is acting on that. Similar way what is of force is going to act on that.

So that is the problems what we will solve it. Second thing is that fluid act as a rigid body motions. Let us take an example here that I have a tank which is a half filled liquid is there. Let us consider it may have water and this tank is accelerated with a constant acceleration a. As you start accelerating this tank with a constant acceleration a, you can understand it, this free surface is going to change it.

And coming to an equilibrium surface such a way that after that this surface will not change it. That means at that time these control volume is moving as a rigid body and traveling with a constant accelerations of a. So we can consider is a very simplified case now, that this fluid control volume what we have here, which is moving with a constant acceleration a, and the with these free surfaces and that what will be there.

And as it is moving it there is no velocity gradient. So no shear stress, so we can adopt the hydrostatic concept. So let me summarize what I am talking it to you that one case we have a gravity dams where we know the fluid is at the rest. That means only the pressure force is going to act on the surface of the dam. In that case we can understand it that there is no velocity, no shear stress, only the pressure what is going to act it.

But in case when you have a rigid body motion, the fluids work as a rigid body motions like the tank is going moving with a constant acceleration of a, the similar conditions can be considered it any liquid field containers moving with the constant accelerations a, and what could be the free surface, what could be the height of the free surface all we can compute it that what in a later on I will tell it.

But the in conceptually in this case what it happens it the control volumes moves at a, constant accelerations. Because of that there is no velocity gradient as that of that there is no shear stress acting it. So we can apply the hydrostatic concept. So one case there is no velocity field at all because fluid is at rest. Another case we have a no velocity gradient. So that is the reasons there is no shear stress.

So we can apply the hydrostatic conditions. That the as equivalent to fluid at the rest conditions.

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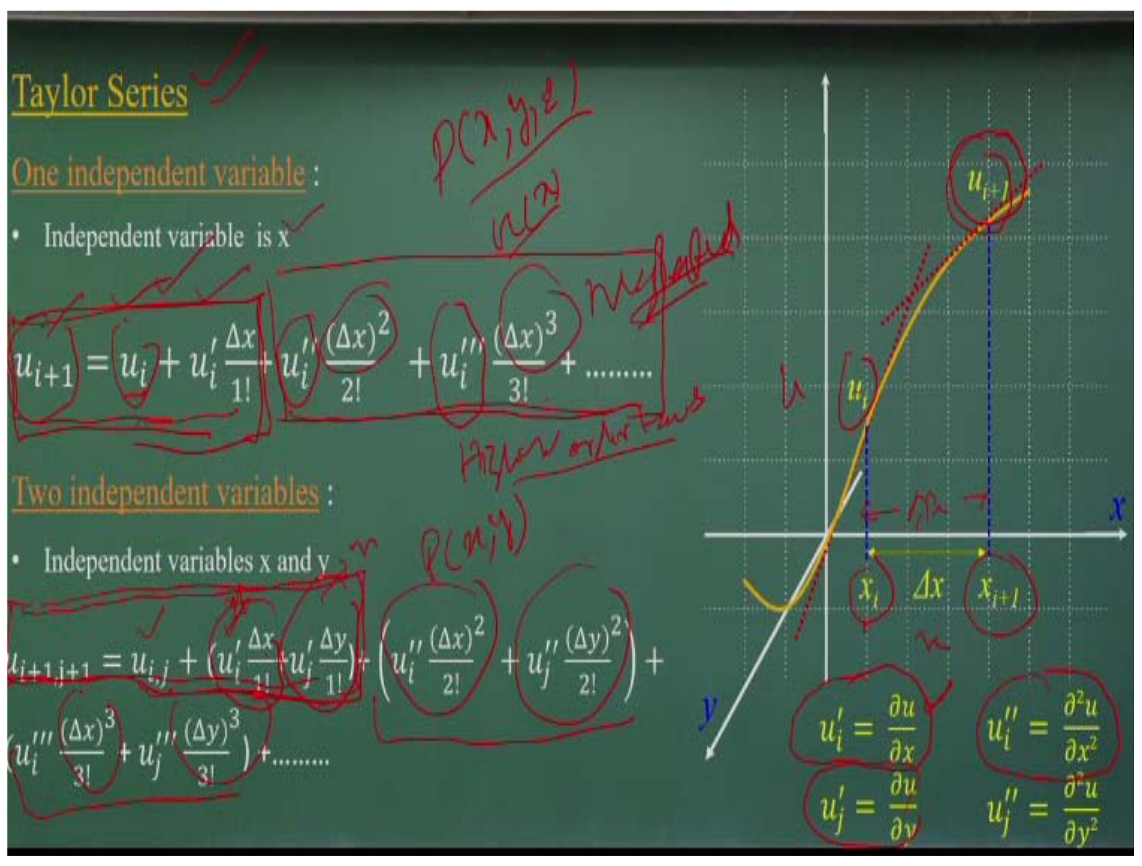

Now whenever I am talking about that I am looking for a pressure field as the fluid is at rest. So basically I am looking at the pressure is a function of the positions $P = P(x, y, z)$. Time is not there, as the fluid is at rest condition. The many of the times when you consider the pressure field, you try to look it from one point to other point. What is that value could be okay?

Like for examples, if I take a very renowned series like a Taylor series, for one independent variables, that means here the u is only a function of x, only the function of the x and u is the variable. And we know what is the value at the x i the u i value, we know that. We want to compute it, what could be the value u_{i+1}which is a Δx distance from x_i and that what is x_{i+1}

So this is a known to us when to compute what could be value u_{i+1}which is having a on the x direction a distance of Δx. The Taylor series gives a infinite series like if you look

$$u_{i+1} = u_i + u_i' \frac{\Delta x}{1!} + u_i'' \frac{(\Delta x)^2}{2!} + u_i''' \frac{(\Delta x)^3}{3!} + \cdots$$

But most of the times what we will look it in a fluid flow problem, we need not need this higher order terms. This is what the higher order terms, okay? We need not need it because these values will be much smaller than this the first term.

So we neglect this high order term, only we consider this part, okay? That is what so this is the higher order terms which are much less than these points. As you can see that $\Delta x^2$is there, Δx^3 is there, and these value becomes much smaller as we go further more higher orders. So we neglect that part. Only we consider any functions if I am approximating at u_{i+1} locations that what will be u_iu i the first gradient of $\frac{\partial u}{\partial x}$, Δx by this value.

That is what I am to say that. So we can approximate either a pressure field the scalar component of velocity field or the density if it is we are considering as the variable in a place and the time component like independent variable of x and y or z or the t. We can define them in a Taylor series, very simple way as if I y_i at a one point y_{i+1} will be the first gradient into Δx. That is the this is what we consider.

This is what we neglect it. This is what we neglected because these terms are very less as compared to the first term. But when you consider two independent variables okay that means the pressure is a functions of x and y for example. In that case we can have the Taylor series is similar way if you can see it that there we will have the i and j :
Independent variables x and y,

$$u_{i+1,j+1} = u_{i,j} + \left(u_i' \frac{\Delta x}{1!} + u_j' \frac{\Delta y}{1!}\right) + \left(u_i'' \frac{(\Delta x)^2}{2!} \quad + u_j'' \frac{(\Delta y)^2}{2!}\right) + (u_i''' \frac{(\Delta x)^3}{3!} + \; u_j''' \frac{(\Delta y)^3}{3!}) + \dots$$

So $y_{i\,j}$ will be again the first order terms of x directions and the y directions. Then again second order term which will be the in x direction and y directions like this we go it. As I already told it, these terms are much lesser because Δx^2we have $\Delta x^3$is there with value as it a square as it a small value this becomes a very less.

So whenever you consider two independent variable case we approximate the function

$$u_{i+1,j+1} = u_{i,j} + \left(u_i' \frac{\Delta x}{1!} + u_j' \frac{\Delta y}{1!}\right)$$

That what we consider when we are applying the Taylor series to approximate a function if I know at initial conditions and the next conditions which is a Δx or Δy from that positions we can approximate the functions like this.

If you are more interested on Taylor series, you can read any mathematical books to know how the Taylor series used for approximating the functions.

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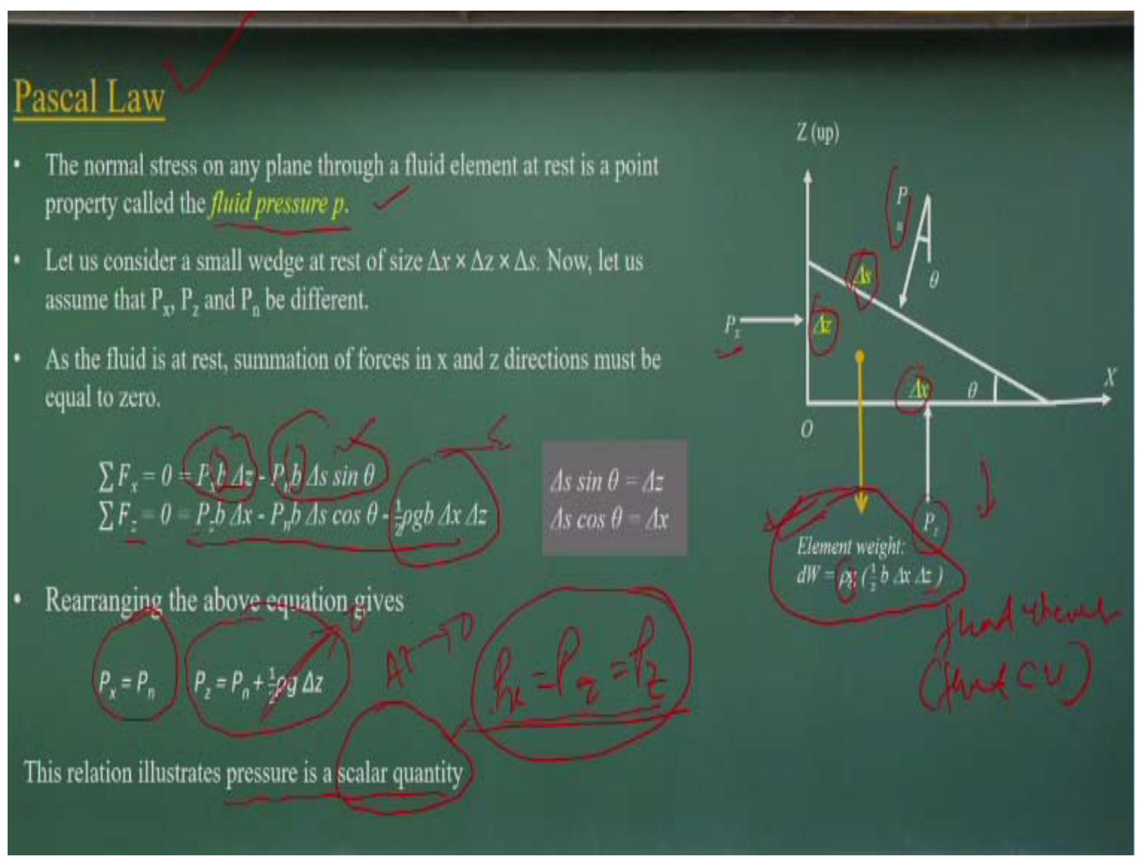

Now let come it to very basic law is called Pascal. As you when you fluid is rest let us consider is that the there will be a normal stress acting on any plane. Okay, that is what we are considering is that that what is the fluid pressure. Here I am considering a fluid element or you can say it the fluid control volume element or fluid control volume.

Now if you consider this is what my control volumes and along this perpendicular to this surface that is what my unit value. That means one unit I have considered perpendicular. This is a three dimensional control volume I am representing as a two dimensional thing. And I am defining the pressure P_x is acting on the surface, the P_j is acting on this surface and P_n is the pressure components acting on this inclined surface. And each one is a Δx, Δz and Δs.

So if it is that when the fluid is rest we can find out some of the forces acting on this control volume should equal to zero. So then we can look it force in a scalar component in x direction and the y direction. That means the force component in x direction should be equal to zero. Force component in the z direction should equal to zero. The force is what? The pressure into the area.

As the fluid is at rest, summation of forces in x and z directions must be equal to zero.

$$\sum F_x = 0 = P_x b\, \Delta z - P_n b\, \Delta s \sin\theta$$

$$\sum F_z = 0 = P_z b\, \Delta x - P_n b\, \Delta s \cos\theta - \tfrac{1}{2}\rho g b\, \Delta x\, \Delta z$$

$\Delta s \sin\theta = \Delta z$ $\Delta s \cos\theta = \Delta x$

What is the weight of these control volumes will be the volume multiplication of unit weight. That is the multiplications of density and acceleration due to the gravity. So you get the element weight that the components if you can see it, that components are here. And we have the force component what you have result. So we have just considered this fluid element or the control volumes at the equilibrium conditions.

Similar way

Rearranging the above equation gives

$$P_x = P_n \qquad P_z = P_n + \tfrac{1}{2}\rho g\, \Delta z$$

That means what you are considering is that your control volume become smaller and smaller, smaller and it could be infinitely smaller value, okay? That what if you consider it z equal to zero then these components become zero. So what will get it?

The $P_x = P_n = P_z$

That means the pressure in the x direction, z direction, and the any direction of normal to the surface will be the same okay? That is what is the Pascal law. And since pressure is same in all the directions, so we can consider is a pressure is a scalar quantity and that what they must stated by Pascal's law.

Very simple way as I consider it a fluid element, fluid at the rest conditions and just equate the pressure due to the pressure the force component and also the gravity component equate in x direction and y direction and consider that Δz tends to the zero. That means your control volumes become smaller and smaller and infinitely smaller. At that periods, you will have a P x, P z and P n are equal.

That is the Pascal law, the mathematically with all its pressure becomes in a fluid at rest condition is a scalar quantity. So fluid flow what we are considering here all the cases the pressure is a scalar quantity. That is what the Pascal's law.

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